UNCERTAINTY ANALYSIS OF STRAIN GAGE CIRCUITS: INTERVAL METHOD AND INTERVAL ALGORITHM

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Abstract- ***This paper presents uncertainty analysis of a strain gage based instrumentation systems. This has been carried out by interval method and classical methods and is verified by the mean value algorithm based on interval arithmetic. The quarter, half and full bridge configuration of strain gage based circuits are considered to illustrate the analysis.***

Index terms: Interval analysis, Uncertainty analysis, Strain gages, Interval Optimization

I.

II. INTRODUCTION

Historically, the development of strain gages has followed many different approaches, and gages have been developed based on mechanical, optical, electrical, acoustical and even pneumatic principles. Electrical resistance strain-gauge nearly satisfies all of the optimum requirements for a strain gage; therefore it is widely employed in stress analysis and as the sensing element in many other applications. The minute dimensional change of mechanical elements in response to a mechanical load, pressure, force, and stress causes a change in the resistance of the strain gage. Wheatstone bridge is commonly employed to convert the resistance change to an output voltage. Although the strain gage is inexpensive and relatively easy to use, care must be exercised to ensure it is properly bonded to specimen, aligned in the direction of measurement, less sensitivity to temperature, and more importantly the lead wire resistance, the excitation source and the accuracy of other components used in the signal conditioning circuit. The widely used strain gage bridge circuit topologies are Quarter bridge, Half bridge and Full bridge configurations [1]. All the strain measuring circuits have some amount of uncertainty associated with them. Understanding the uncertainty within our predictions and decisions is at the heart of understanding the problem. Uncertainty analysis using classical methods for electrical and electronic circuits can be seen in [2, 3, 4]. Uncertainty analysis using interval arithmetic is more reliable and it does not use statistical methods and it can handle simultaneously the uncertainty in more than one parameter. In interval method, the uncertain parameters are assumed to be unknown but bounded and each of them has an upper and lower limit without a probabilistic structure. As uncertainty information required for the interval method is lesser, it happens to be an attractive prospect for engineering applications. It is an alternative and valid technique to compute how the system accuracy varies with the variation in parameters and the interval methods are able to prove (or disprove) with mathematical rigor, the existence of desired solutions. Interval methods have been used for the uncertainty analysis of passive and active electric circuits, power cables, civil and mechanical structures [5, 6, 7, 8, 9]. However the application of this technique to instrumentation systems has not been attempted. In this paper, the uncertainty analysis of strain gage circuits using interval and classical methods is carried out.

III EXPERIMENTAL SET UP

The experimental setup to measure the strain in a cantilever beam made of aluminum is shown in Figure 1. The strain at the fixed end of the beam is measured using three different strain measuring circuits namely, quarter bridge, half bridge and full bridge and are shown in Figure 2. The measuring circuits are excited with amplitude of 5 volts using CA 100 Yokogowa universal calibrator. The resistance of strain gage is 350 ohms, the resistance of fixed resistors used in the measuring circuit is 350 ohms and the resistance of lead wire connecting the strain gage and the measurement circuit is 1.21 ohms. The tolerance of the excitation source is ± 0.0025 volts, that of fixed resistors are ± 10 % and that of lead wire resistance is ± 1 %.

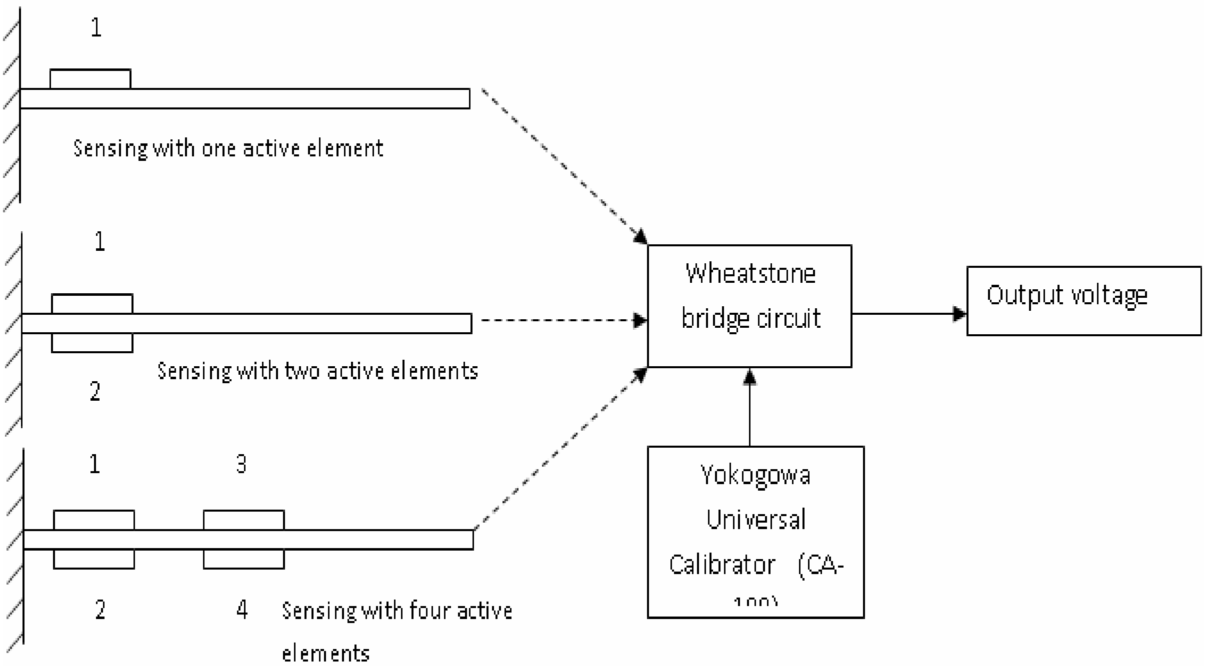
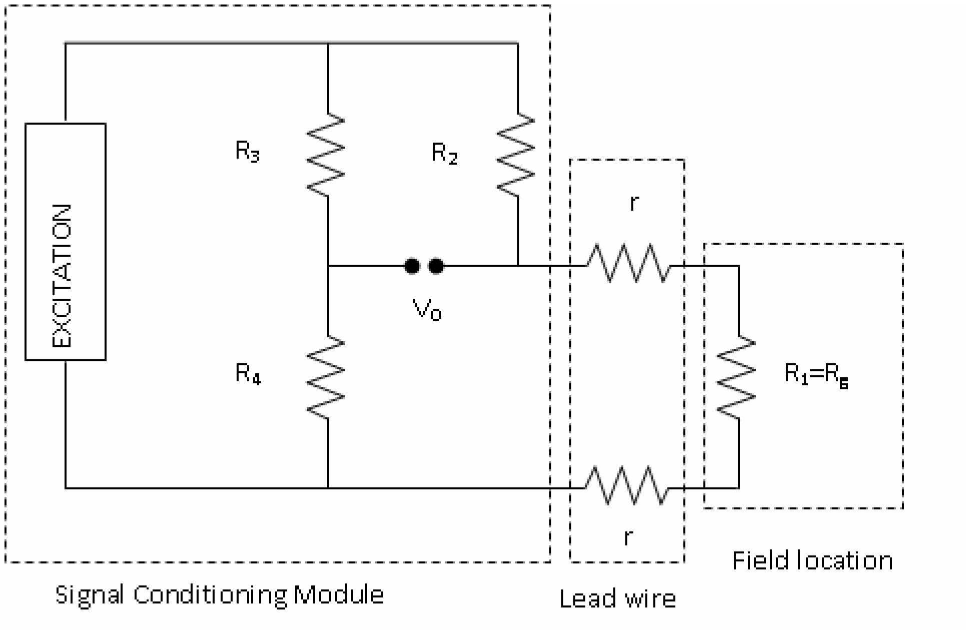


Figure 1 Schematic of experimental setup for strain measurement


EXCITATION
R3
R2
r
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R4
R1=RG
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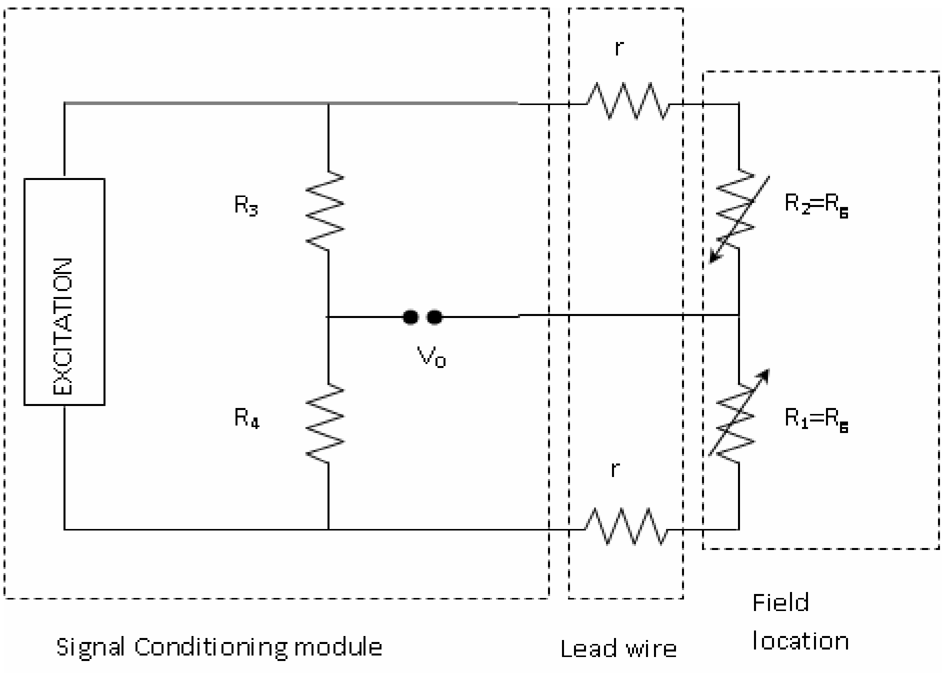
Signal Conditioning Module
Lead wire
Field location

(a)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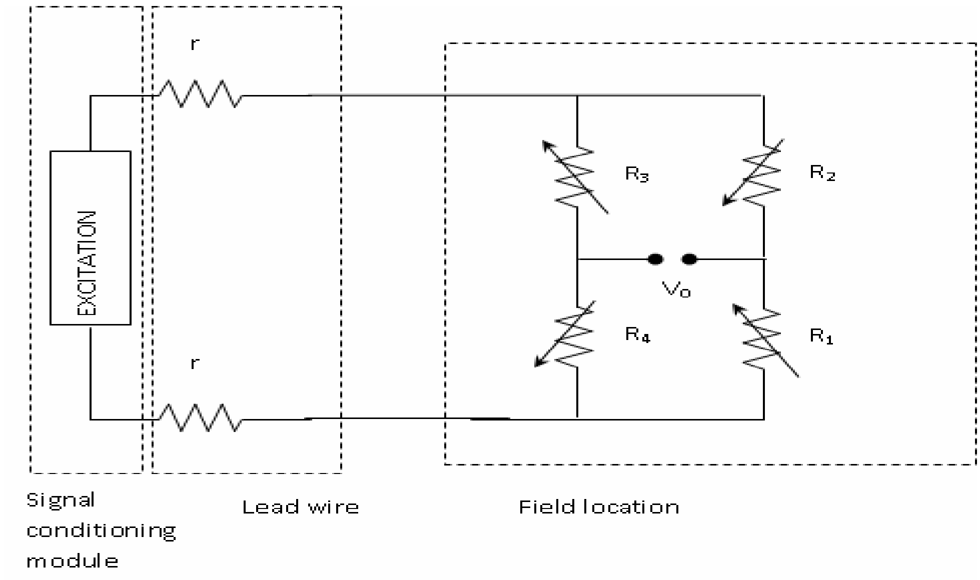

(c)

Figure 2 Strain Measuring circuits

(a) Quarter bridge (b) Half bridge (c) Full bridge

IV ANALYSIS USING INTERVAL ARITHMETIC

In strain measurement, the uncertainty can arise from the process, strain gage, measuring circuits, lead wire and data representation element. In comparison to the classical methods, interval method considers all the sources of uncertainty and estimate in a single step of evaluation [10, 11]. Hence it is proposed that interval method is a viable and alternative tool for uncertainty analysis of strain gage measuring circuits.

Quarter bridge arrangement shown in Figure 2a utilizes a single active strain gage in position R_1 and is often employed for both static and dynamic strain measurements if the temperature compensation is not required. The resistance $R_1 = R_G$ and the other three resistances are selected to maximize the circuit sensitivity while maintaining the balance condition $R_1R_3 = R_1R_4$. The performance function with the lead wire resistance (r) is

$$V_0 = \left(\frac{\frac{[V_{in}]}{4} \times \frac{\Delta R_G}{[R]}}{\left(1 + \frac{\Delta R_G}{2[R]} + \left(\frac{[r]}{[R]} \times 2 + \frac{\Delta R_G}{2[R]}\right)\right)} \right) \tag{1}$$

The half bridge arrangements shown in Figure 2b, utilizes two active strain gages in position R_1 and R_2 and are denoted as R_G and the performance function is

$$V_0 = \left(\frac{[V_{in}] \times \frac{\Delta R_G}{[R]}}{\left(2(1 + \frac{2[r]}{[R]} \right)} \right) \tag{2}$$

In full bridge configuration, four active strain gages are used as shown in Figure 2c. When the gages are placed on a cantilever beam in bending, with tensile strain on gages 1 and 3 (top surface of the beam) and compressive strain on gages 2 and 4 (bottom surface of the beam), the performance function with lead wire resistance (r) is

$$V_0 = \left(\frac{[V_{in}] \times \left(\frac{\Delta R_G}{[R]} \right)}{(1 + \frac{2[r]}{[R]})} \right) \tag{3}$$

where Vo is the output voltage, V_{in} is the excitation voltage, the resistance of fixed resistors are denoted as $R_2 = R_3 = R_4 = R$, 'r' is the lead wire resistance and ΔR_G is the change in resistance of stain gage. The performance function of a quarter, half and full bridge strain measuring circuit given in equations (1), (2) and (3) are expressed in the form of natural interval extension function by replacing the uncertain input parameters in interval form. The interval form of uncertain input parameters are Vin = [4.9975, 5.0025] volts, R = [315, 385] ohms and r = [1.1979, 1.2221] ohms. The output voltage of quarter, half and full bridge circuits for the input of 0.1kg mass are computed in interval form as [0.6448, 0.7849] mV, [1.2868, 1.5808] mV and [2.5787, 3.1516] mV respectively [12]. The relative uncertainty for all three measuring circuits is given in Table 1.

Table 1 Relative uncertainty of strain measuring circuits

Measuring circuits	Nominal output voltage (mV)	Output voltage in interval form(mV)	Mid-point of interval (mV)	Radius of interval (mV)	Relative Uncertainty
Quarter Bridge	0.7084	[0.6448, 0.7849]	0.7149	0.0701	0.09806
Half Bridge	1.4188	[1.2868, 1.5808]	1.4338	0.1470	0.1025
Full Bridge	2.8375	[2.5787, 3.1516]	2.8652	0.2865	0.0999

V ANALYSIS USING MEAN VALUE FORM ALGORITHM

The mean value form of optimization algorithm is used to analyze the uncertainty in strain measuring circuits. It is the branch and bound operation of interval optimization algorithm, which is based on box generation, split and delete. Box generation is to represent an interval number by a series of smaller sub boxes. The lower and upper bound of sub box series should be lower and upper bound of interval number respectively.

Mean value form is a particular form of interval extension which is applicable to arbitrary functions with continuous first order derivatives. Let $f : R^n \rightarrow R$, X is an interval vector and m(X) is mid-point of the interval vector. For any $y \in X,$ the mean value theorem states that

If $f_j^{'}$ denotes the interval extension of $\frac{\partial f}{\partial x_i} = f_j^{'}$ on X, then

$$f(y) = f(m) + \sum_{i=1}^{n} F_j^{'}(X)\left(X_i - m_i\right), \text{for any } y \in X. \tag{4}$$

The right hand side of the equation (4), is called the mean value extension of f on X and it is given as

$$F_{MV}(X) = f(m) + \sum_{i=1}^{N} G_i(X)\left(X_i - m_i\right) \tag{5}$$

The mean value form is inclusion monotonic and assures better interval extension as compared to the natural extension function $f(x)$ for narrow enough intervals X. In the global optimization process, the performance function $f(x)$ is formulated into two unconstrained global minimization problems as

$$f_L^* = \min f(x), \; x \in x^0$$
$$f_U^* = \min\, (-f(x)), \; x \in x^0, \text{where}$$

f_L^* provides the lower end point of the performance function $f(x)$

f_U^* provides the upper end point of the performance function $f(x)$

The f_L^* and f_U^* are computed based on the mean value form given in equation (5) by following the algorithm steps.

Step 1: Break the given interval of uncertain input parameters (excitation source, fixed resistor and lead wire resistance of measurement system) into sub boxes of smaller width of equal sizes and are placed in a list L

Step 2: At each iterations of the algorithm, extract the sub box which has a smallest lower bound from the list L

Step 3: Compute $F(x)$ based on mean value form given in equation (5), where F(x) are the equations (1), (2) and (3). And set the $F(x)$ as [F_L, F_U]

Step 4: Compute $f(m_i)$ for $i = 1, 2, 3$ using equations (1), (2) and (3).

Step 5: Take an arbitrary point $x_{0_i}\ for\ i = 1,2,3$ somewhere around the midpoint of each input uncertain parameters, compute $f(x_{0_i})$.

Step 6: Compare F_L and $f(x_{0_i})$. If $F_L > f(x_{0_i})$, then delete the current sub box from the list L. After deleting it take the next sub box with smallest lower bound value and continue from step 3. Else check whether $f(x_{0_i}) - F_L <= e$, where 'e' is the desired accuracy. If the above condition is true, then the current sub box is the final range of $f(x_i)$ for $i = 1, 2, 3$ and the algorithm is terminated. If the above condition is false go to the next step.

Step 7: In this step, the size of the current sub box is tried to be reduced in x_1 direction. Compute Y' from

$$G_1(X)Y' + G_2(X)(X_{c1} - X) \le e$$

$$\text{where } G_1(X) = \frac{\partial f}{\partial x_1} \text{ and } G_2(X) = \frac{\partial f}{\partial x_2},\ X_{c1} = \text{current sub box}$$

Set the resulting set as Z_1. Compute the desired set Y_1 as, $Z_1 \cap X_1$. Next try to reduce X in the x_2 direction using Y_1 rather than X_1. Compute Y'' from the equation given

$G_2(X)Y'' + G_1(X)(Y_1 - X) \le e$, and put the corresponding set as Z_2. Compute the desired set Y_2 from $Z_2 \cap X_2$. Computations process continues in a similar way until all input uncertain parameters Y_i of Y are determined.

Step 8: Compute $F(Y_i)$ i =1, 2, 3 and put the lower limit of the resulting operation as $F_L(Y_i)$

Step 9: If $F_L(Y_i) < f(x_{0_i})$, then update $f(x_{0_i})$ by $F_L(Y_i)$

Step 10: Now divide the current sub boxes into smaller sub boxes and enter the values of these sub boxes into the list L replacing the older values.

Step 11: Continue again from step 1 until the desired accuracy is reached.

For the three strain measuring circuits, all the uncertain input parameters in the interval form are further divided into small sub boxes. Moore's uniform partitioning technique is used such that each uncertain parameter of circuit is divided as $X=[\overline{x},x_1] \cup [x_1,x_2] \cup [x_2,x_3] \cup ....[x_{p-1},\underline{x}]$

The list L is formed with all small sub boxes of the uncertain parameters. Here the selection of sub box from the list L is based on Moore- Skelboe's approach, i.e the sub box which has the lowest lower bound value, has to be extracted for further computation. The number of sub boxes for each uncertain parameter in strain measuring circuits is taken as twenty five. The range of output voltage of the three strain measuring circuits for 0.1kg of mass placed at the free end of the cantilever beam is computed for two different accuracies.

Generally, the sub boxes of interval number will have the same width after splitting. In this work, it is proposed to split the interval into smaller subboxes with equal and unequal width. It is also proposed to use subboxes with narrow width around the nominal value and to use the subboxes with wider width near the bound of the interval [13, 14]. The results for equal and unequal sub-division of uncertain parameters for the accuracy of 0.001 and 10^{-10} are given in Table 2 and 3 respectively.

Table 2 Uncertainty in output voltage using mean value form
(For accuracy of 0.001, number of subboxes 25)

	Measuring circuit	Range of input uncertain parameters			Range of output voltage (V_0)
		V_{in} (V)	R (Ohms)	r (Ohms)	
Equal subdivision of input uncertain parameters	Quarter bridge	[4.99750, 4.997600]	[348.9996, 352.9923]	[1.19701, 1.19884]	[0.70950, 0.71155]
	Half bridge	[4.997500, 4.997604]	[349.94980, 352.89994]	[1.19780, 1.19779]	[1.41140, 1.42320]
	Full bridge	[4.99750, 4.99704]	[349.94979, 352.89894]	[1.19780, 1.19779]	[2.8123, 2.8368]
Unequal subdivision of input uncertain parameters	Quarter bridge	[4.99501, 4.99772]	[347.2394, 353.2199]	[1.19698, 1.19899]	[0.70970, 0.71363]
	Half bridge	[4.99750, 5.00186]	[349.8990, 325.7612]	[1.19980, 1.2011]	[1.4234, 1.4310]
	Full bridge	[4.99750, 5.00165]	[349.94978, 353.99890]	[1.19780, 1.19779]	[2.80671, 2.83690]

Table 3 Uncertainty in output voltage using mean value form
(For accuracy of 10^{-10}, number of subboxes 25)

	Measuring circuit	Range of input uncertain parameters			Range of output voltage (V_0)
		V_{in} (V)	R (Ohms)	r (Ohms)	
Equal subdivision of input uncertain parameters	Quarter bridge	[4.99752, 5.00121]	[347.2130, 353.1121]	[1.19702, 1.19889]	[0.70892, 0.70993]
	Half bridge	[4.99750, 5.00163]	[349.94980, 354.01231]	[1.19700, 1.19780]	[1.40810, 1.42320]
	Full bridge	[4.99750, 5.00165]	[349.94978, 353.99891]	[1.19700, 1.19780]	[2.80661, 2.83731]
Unequal subdivision of input uncertain parameters	Quarter bridge	[4.99501, 5.00131]	[348.8173, 353.78956]	[1.19701, 1.19898]	[0.70899 0.71003]
	Half bridge	[4.99851, 5.00223]	[350.8990, 353.0213]	[1.19780, 1.2010]	[1.41210, 1.41950]
	Full bridge	[4.997504, 5.00186]	[349.94971, 351.02132]	[1.19780, 1.19779]	[2.80668, 2.83730]

VI ANALYSIS USING CLASSICAL METHODS

(a) Worst case analysis

Worst case analysis or tolerance analysis is the method of analyzing a piece of a design using the high and low ends of the tolerance spread for each parameter/variable. This extreme-case investigation allows designers to predict whether the designed system will stay within its desired performance limits under all the possible combinations of parameter variation. The main objective for this is to prescribe safety margins in the sensitive areas of system design so that reliability is incorporated into the hardware for long term trouble-free field operation. Worst case analysis also determines the mathematical sensitivity of system's performance to these variables and provides both statistical and non-statistical methods for handling the variables that affect the system. One of the basic limitations of worst case analysis is that it does not give a unique solution to the problem since it uses only the first-order Taylor series for the analysis, neglecting the nonlinear terms. Further, it is possible that there might be several combinations of individual errors that give the same overall uncertainty in the output variable. Also, computationally this could lead to a combinatorial explosion in large-scale systems [15].

In this work, the variation in the output voltage of the strain measuring circuits are computed using first order Taylor series expansion as

$$\left|\Delta Vo\right| = \left|\frac{\partial Vo}{\partial V_{in}}\right|\left|\Delta V_{in}\right| + \left|\frac{\partial Vo}{\partial R}\right|\left|\Delta R\right| + \left|\frac{\partial Vo}{\partial r}\right|\left|\partial r\right| \tag{6}$$

where $\left|\Delta Vo\right|$ is the cumulative variation in the output voltage due to the individual component tolerances. The output voltage ($V_{0nominal}$) of the quarter, half and full bridge circuits are measured using equations (1), (2) and (3) as 0.7084 mV, 1.4188 mV and 2.8375 mV respectively for 0.1 kg of mass placed at the free end of the cantilever beam. The variation of the output voltage $\left|\Delta Vo\right|$ of the quarter, half and full bridge circuits are 0.1431 mV, 0.2834 mV and 0.5668 mV respectively. The range of output voltage of the quarter, half and full bridge circuits with uncertainty are (0.5653 to 0.8515 mV), (1.3540 to 1.7022 mV) and (2.2707 mV to 3.4043 mV) respectively.

(b) Method of moments

It is a statistical technique of parameter estimation, where the probabilistic moments of the distributions of uncertain parameters are equated with the sample moments, and the unknown parameter is estimated. This method of moments, in general, provides estimators which are consistent, but not as efficient as the maximum likelihood ones. Moreover, this technique results in simpler computations, unlike the maximum likelihood method which can be computationally cumbersome. The following are the limitations of this method: (i) any moment of an uncertain parameter provides a summary of its distribution with the loss of resolution, (ii) exact selection of probability distribution of uncertain parameter is very difficult, and (iii) even a small deviation form the real probability distribution may cause large error in system.

In this paper, the nominal value, the variance and the uncertainty in the output voltage of the quarter, half and full bridge strain measuring circuits are computed as

$$\mathrm{var}\,(V_0) \approx \sum_{i=1}^{3} b_i^2\, \sigma_i^2$$
$$\text{where } b_i = \frac{\partial V_0}{\partial x_i},\; x_1 = V_{in},\; x_2 = R,\; x_3 = r \text{ and } \sigma_i = \left(\frac{x_{i-\max} - x_{i-\min}}{6} \right) \tag{7}$$

The variance of the output voltage of quarter, half and full bridge circuits are computed using equation (7) as 0.0166 mV, 0.0702 mV and 0.0993 mV respectively. The range of output voltage of quarter, half and full bridge circuits with uncertainty is computed as (0.6918 to 0.7250 mV), (1.3486 mV to 1.4890 mV) and (2.7382 mV to 2.9368 mV) respectively.

(c) Monte Carlo method

Monte Carlo method is a sample based method. Primarily, it provides a complete frequency distribution of the output variable for a randomly chosen (uncertain) input variable. It thus gives a global view for uncertain parameter combinations. Accuracy may be increased at the cost of larger sample size and hence, demands computational resources. In principle, this technique uncovers some of the potentially error-producing combinations found in the worst-case analysis. This method finds ready application in most of the instrumentation applications for reasons such as the following: (i) the probability distributions of the parameter variations within the model of the system can be easily and flexibly modeled, without the need to approximate them, (ii) corrections and the other relations, dependencies can be modeled without difficulty and the investigation can be made with great ease and speed, and (iii) the level of mathematics required is quite basic and commercial simulation packages can automate the tasks involved in simulations. Accuracy of the solution at the cost of computational resources is basic disadvantage of this method.

In this paper, the number of trails is fixed as 1000 and the uncertain input parameters namely, excitation source, resistance of fixed resistor and the lead wire resistance (r) of the all the three strain measuring circuits are assumed to have uniform distribution with a confined limit of tolerance levels. The range of output voltage of f quarter, half and full bridge circuits for the input of 0.1kg of mass applied at free end of the beam is (0.6453 to 0.7868 mV), (1.2912 mV to 1.5746 mV) and (2.5461 mV to 3.1506 mV) respectively.

VII. CONCLUSIONS

The performance functions of quarter, half and full bridge strain measuring circuits are non inclusive hence, the narrow width for output voltage of these circuits is obtained by method of moments instead of interval arithmetic. It is found from the analysis using mean value form that, with equal width subboxes around the nominal value and unequal width subboxes around the bounds produces the same result as that of unequal subboxes in the entire range. This suggests that the division of subboxes need not be selected equal and the entire range can be divided into unequal subboxes or combinations of equal and unequal subboxes. As the accuracy increases, the width of the output voltage decreases. The results for two different accuracy level using mean value form indicates that for the quarter bridge, all three input uncertain parameters (V_{in}, R, r) have to be tightly controlled and the excitation source (V_{in}) needs to be tightly controlled for half and full bridge circuits. The results in Table 2 and 3 are obtained by having the number of subboxes as 25 for the accuracy of 0.001 and 10^{-10}. The algorithm is also executed for 10 subboxes for the same accuracies and it is found that as the number of subboxes and accuracy increases, the number of iterations required is also increases.

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